

**Activity 2**

Here are two questions from the June Pure 2 paper

- 3. (i)** Use algebra to prove that for all real values of  $x$

$$(x - 4)^2 \geq 2x - 9 \quad (3)$$

- (ii)** Show that the following statement is untrue.

$$2^n + 1 \text{ is a prime number for all values of } n, n \in \mathbb{N} \quad (1)$$

- 8. (i)** Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form  $a + b \log_2 3$ , where  $a$  and  $b$  are constants to be found. **(4)**

- (ii)** Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2 \log_5(y + 1) - 1 \quad (5)$$

Use the mark scheme to mark the following answers:

A

(1)

i)  $(x-4)(x-4)$   $x^2 - 4x - 4x + 16$   
 $x^2 - 8x + 16$

$x^2 - 8x + 16 \geq 2x - 9$   
 $x^2 - 10x + 25 \geq 0$   
 $(x-5)^2 \geq 0$

$x = 5$

$5 \geq 0$

ii)  $2^n + 1$

$2^1 + 1 = 3$  True

$2^2 + 1 = 5$  True

$2^3 + 1 = 9$  False

$2^4 + 1 = 17$  True

$2^5 + 1 = 33$  False

The statement is not true the equation doesn't result a prime number

$$i) \quad (x-4)(x-4) \geq 2x-9$$

$$x^2 - 4x - 4x + 16 \geq 2x - 9$$

$$x^2 - 8x + 16 \geq 2x - 9$$
$$-2x \quad +9 \quad -2x \quad +9$$

$$x^2 - 10x + 25 \geq 0$$

$$\left(x - \frac{10}{2}\right)^2 - \left(\frac{-10}{2}\right)^2 + 25 \geq 0$$

$$(x-5)^2 - 25 + 25 \geq 0$$

$$(x-5)^2 \geq 0$$

$(x-5)^2$  gives all ways positive value for any number or gives zero if  $x$  is zero so all values will be greater or equal to zero.

ii) even = 6 ~~odd~~ odd = 3

$$2^6 + 1 = 65$$

not  
prime

$$2^3 + 1 = 9$$

not prime

$$\text{ii) } (x-4)(x-4) \geq 2x-9$$

$$x^2 - 4x - 4x + 16$$

$$x^2 - 8x + 16 \geq 2x - 9$$

$$-2x \quad -2x$$

$$x^2 - 10x + 25 \geq 0$$

$$(x-5)$$

$x=5$  is a real value

all are real roots

$$\text{ii) } 2^1 + 1 = 3$$

$$2^2 + 1 = 5$$

when <sup>the power</sup> ↑ gives an even number

(+1) gives us the odd

power gives an even number

so the (+1) change it to

an prime number / odd number

8i)

$$2x + 1 \log_2 8 = \log_2 6$$
$$(2x + 1) 3 = \log_2 6$$
$$6x + 3 = \log_2 6$$

8i)  $(2^3)^{2x+1}$

~~$\log 8$~~

$$\log_8 6 \cdot 2x + 1$$
$$2x + 1 = \frac{\log 6}{\log 8}$$

$$2x + 1 \log_8 8 = \log_2 6$$

$$2x + 1 + 1 = \log_2 6$$

$$2x + 2 = \log_2 6$$

$$2$$

$$x + 1 = \log_2 3$$

$$\frac{\log(2 \times 3)}{\log(2^3)}$$

$$\frac{1 + \log_2^3}{\log_2^8}$$

$$\frac{1 + \log_2^3}{2}$$

$$\frac{1}{3} + \log_2^3$$

$$\boxed{\frac{1}{3} + \log_2^3}$$

$$a = \frac{1}{3} \quad b = 1$$

→

$$\text{ii) } \log_5 (7 - 2y) = 2 \log_5 (y + 1) - 1$$

$$\log_5 \left( \frac{7 - 2y}{(y + 1)^2} \right) = -1$$

$$(y + 1)(y + 1)$$

$$y^2 + 2y + 1$$

$$\frac{1}{5} = \frac{7 - 2y}{y^2 + 2y + 1}$$

$$y^2 + 2y + 1 = \begin{array}{r} 35 - 10y \\ -35 + 10y \end{array}$$

$$y^2 + 12y - 34 = 0$$

~~$$y = -6 \pm \sqrt{40}$$~~

$$y = -6 + \sqrt{70}$$

$$y = -6 - \sqrt{70}$$



$$i) (2^3)^{2x+1} = \log 6$$

$$6x + 3 = \log 6$$

$$6x + 3 = \log 6$$

$$ii) \log_5(7-2y) = 2\log_5(y+1) - 1$$

$$\log_5(7-2y) - 2\log_5(y+1) = -1$$

$$\log_5 \frac{(y+1)^2}{7-2y} = -1$$

$$\frac{y^2 + 2y + 1}{7-2y} = -1$$

$$(7-2y) \times -1 = y^2 + 2y + 1$$

$$-7 + 2y = y^2 + 2y + 1$$

$$\begin{array}{r} -7 + 2y = y^2 + 2y + 1 \\ \quad \quad \quad -2y \quad \quad \quad -2y \\ \hline y^2 + 8 \end{array}$$

$$y^2 = 8$$

$$y = 2\sqrt{2} = 2.83$$

i.  $2^{3(2x+1)} = 6$        $2^{6x} \times 2^3 = 6$

~~2^{6x} = \frac{6}{8}~~       $2^{6x} = \frac{3}{4}$

~~6x \log 2 = \log \frac{3}{4}~~       $6x \log 2 = \log \frac{3}{4}$

~~6x = \frac{\log \frac{3}{4}}{\log 2}~~       $6x = -0.415$

~~x = \frac{-0.415}{6}~~       $x = -0.069$

  

ii.  $\log_5(7-2y) - \log_5(y+1)^2 = -1$

~~$\log_5 \frac{7-2y}{(y+1)^2} = -1$~~

$\log_5 \frac{7-2y}{(y+1)^2} = -1$

$\frac{7-2y}{(y+1)^2} = \frac{1}{5}$

$35 - \log y = y^2 + 2y + 1$

$y^2 + 12y - 34 = 0$

~~$y = -6 \pm \sqrt{70}$~~

$y = -6 + \sqrt{70}$

$$i) (8^2)^x * 8 = 6$$

$$8 = 2^3$$

$$(2^6)^x * 8 = 6$$

$$(2^6)^x * 2^3 = 6$$

$$2^{6x+3} = 6$$

~~$$\log_2 8 = 3$$~~

$$\log_2 6 = 6x+3$$

~~$$3 \log_2 2$$~~ 
$$\log_2 2 + \log_2 3 = 6x+3$$

$$1 + \log_2 3 = 6x+3$$

$$\log_2 3 = 6x+2$$

$$\frac{\log_2 3 - \frac{1}{3}}{6}$$

$$\frac{1}{6} \log_2 3 - \frac{1}{18}$$

$$-\frac{1}{18} + \frac{1}{6} \log_2 3$$

$$\text{ii) } \log_5 (7-2y) - 2 \log_5 (y+1)^2 = -1$$

$$\log_5 \left( \frac{7-2y}{y^2+2y+1} \right) = -1$$

$$5^{-1} = \frac{7-2y}{y^2+2y+1}$$

$$\frac{1}{5} y^2 + \frac{2}{5} y + \frac{1}{5} = 7-2y$$

$$\frac{1}{5} y^2 + \frac{12}{5} y - \frac{34}{5}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-\frac{12}{5} \pm \sqrt{\left(\frac{12}{5}\right)^2 - 4 \times \frac{1}{5} \times -\frac{34}{5}}}{2 \times \frac{1}{5}}$$

$$y = -6 + \sqrt{70} = 2.4$$

$$y = -6 - \sqrt{70} = -14.4 \quad \times$$

$$\boxed{y = 2.4}$$

because  $-14.4$  is rejected <sup>since in</sup>  $2 \log_5 (-14.4+1)$   
is a Math error

- 5 120 children go on an activity holiday.  
The ratio of the number of girls to the number of boys is 3:5

On Sunday, all the children either go sailing or go climbing.

$\frac{16}{25}$  of the boys go climbing.

Twice as many girls go sailing as go climbing.

Work out how many children go sailing on Sunday.

$$3:5 = 120$$

$$45:75 \quad B = \frac{16}{25} \text{ of } 75 = 48$$

$$G = 2:1 = 45$$

$$G = \frac{45}{3}$$

$$B = \begin{array}{r} 75 \\ - 48 \\ \hline 27 \end{array}$$

$$G = 30:15$$

$$\begin{array}{r} 27 \\ + 30 \\ \hline 47 \end{array}$$

47

(Total for Question 5 is 6 marks)

- 5 120 children go on an activity holiday.  
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Twice as many girls go sailing as go climbing.

Work out how many children go sailing on Sunday.

$$\text{girls} = 3$$

$$\text{boys} = 5$$

$$\text{boy} = \frac{5}{8} \times 120 = 75$$

$$\text{girl} = \frac{3}{5} \times 120 = 45$$

$$\text{boys climbing} = \frac{16}{25} \times 75 = 48$$

$$\text{girls} = 24$$

$$48 + 24 = 72$$

72

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(Total for Question 5 is 6 marks)

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Twice as many girls go sailing as go climbing.

Work out how many children go sailing on Sunday.

$$3+5=8 \rightarrow 120 \div 8 = 15$$

$$15 \times 3 = 45, 15 \times 5 = 75$$

$$75 \div 25 = 3 \rightarrow +16 = 19$$

$$75 - 19 = 56$$

~~$$3 \times 8 = 24$$~~

✓ Sailing (8)

~~$$\div 8 = 5$$~~

$$120 - 56 = 64$$

$$\downarrow \div 2 = 32 \rightarrow \frac{32}{2} = 16$$

↓  
Sailing (6)

↓  
Climbing (6)

$$32 + 56 = 88$$

88

(Total for Question 5 is 6 marks)